Faculty of Civil Engineering Belgrade
Master Study COMPUTATIONAL ENGINEERING
Fall semester 2005/2006

ASSIGNMENTS

LESSON V

Eigensystems

[**1**] Determine the characteristic polynomial of matrix

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

Use

a) Krilow's method and Gauss method for final system of linear equations,
b) SSP subroutines (Scientific Subroutine Package - IBM) `NROOT` and `EIGEN`,
c) System `Mathematica`.

(Solution: $P(\lambda) = \lambda^4 - 4\lambda^3 + 2\lambda^2 + 5\lambda + 2$).

[**2**] Using system `Mathematica` compute eigenvalues and eigenvectors of matrix $A(\theta)$.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

[3] Write procedures in `Fortran/Pascal/Mathematica/C` for determination of characteristic polynomial of matrix

$$\begin{bmatrix} 15 & 11 & 6 & -9 & -15 \\ 1 & 3 & 9 & -3 & -8 \\ 7 & 6 & 6 & -3 & -11 \\ 7 & 7 & 5 & -3 & -11 \\ 17 & 12 & 5 & -10 & -16 \end{bmatrix},$$

using Krilow's and Leverier's method. Polynomial roots (eigenvalues) compute by using `SSP` subroutine `POLRT`. Write also a complete procedure in `Mathematica`. (Solution: $p(\lambda) = \lambda^5 - 5\lambda^4 + 33\lambda^3 - 51\lambda^2 + 135\lambda + 225.$)

[4] Determine eigenvalues and eigenvectors of matrices

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

[5] Determine eigenvalues and eigenvectors of matrix

$$\begin{bmatrix} 0 & 5 & 8 \\ 5 & 0 & 8 \\ 8 & 5 & 0 \end{bmatrix}$$

using arbitrary method. Check the result by `NROOT` and `EIGEN` subroutines of `SSP` package.

[6] For the matrix

$$\begin{bmatrix} 1/3 & 1 & 1 \\ 1/4 & 2/3 & 1/2 \\ 1/5 & 1/2 & 1/3 \end{bmatrix}$$

define three discs containing eigenvalues and then compute eigenvalues.

[**7**] Determine eigenvalues and eigenvectors of matrix

$$\mathbf{A} = \begin{bmatrix} 0 & a & a^2 \\ a^{-1} & 0 & a \\ a^{-2} & a^{-1} & 0 \end{bmatrix}.$$

[**8**] For eigenproblem

$$\begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

a) form the characteristic equation and compute eigenvalues,
b) compute eigenvectors.

[**9**] Find eigenvalues and eigenvectors of matrix

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

For characteristic polynomial use method of Leverrier and for polynomial roots subroutine `NROOT` of `SSP` package.

[**10**] Find a modal matrix $\mathbf{X}$ of matrix

$$\mathbf{A} = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix},$$

where $\mathbf{X}^{-1} \cdot \mathbf{A} \cdot \mathbf{X} = \mathbf{\Lambda}$, and

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}.$$

[11]* For a simple plane frame of arbitrary dimension form a stiffness matrix using program `STRESS`. Then calculate eigenvalues and eigenfrequencies by formula

$$\omega = \sqrt{\lambda_i}.$$

By simple change in frame dimensions shift basic (lower) frequency up or down. Reduce the higher frequencies by simple constructive change.

[12] Determine eigenvalues and eigenvectors of a matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix},$$

by arbitrary method.

[13] Proceed Jacobi method on the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

by hand and using program `Mathematica`.

[14] Proceed the Given's method on the Hilbert matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}.$$

[15] Find eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} 17 & -1 & -27 & -6 \\ 6 & -14 & -54 & -24 \\ 1 & 1 & -29 & -4 \\ -9 & -19 & 51 & 6 \end{bmatrix}.$$

Solution: Characteristic function is

$$\lambda^4 + 20\lambda^3 - 700\lambda^2 - 8000\lambda + 120000,$$

with eigenvalues

$$-30 \qquad -20 \qquad 10 \qquad 20$$

and eigenvectors

$$\begin{bmatrix} 87 \\ 546 \\ 91 \\ 181 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -1 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \\ 1 \\ -9 \end{bmatrix}.$$

[**16**] Calculate characteristic function of a matrix

$$\mathbf{A} = \begin{bmatrix} 17 & 17 & 27 & 12 \\ 6 & -14 & -54 & -24 \\ 1 & 1 & -29 & -4 \\ -9 & -19 & 51 & 6 \end{bmatrix}$$

using Leverrier's method.
Solution: Characteristic function is

$$\lambda^4 + 20\lambda^3 - 700\lambda^2 - 8000\lambda + 120000.$$